Quasi Orthogonal Space Time Block Codes Using Various Sub-Block Matrices

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***Abstract*—Space-time block codes (STBCs) from Orthogonal designs have attracted attention due to their fast maximum likelihood decoding and full diversity, but the maximum symbol transmission rate for complex signals is only ¾ for four transmit antennas so quasi orthogonal STBC was proposed, in this paper the structure and characteristics of some new type of a Quasi-orthogonal space time block codes (QOSTBC)is analyzed, The error performance for these codes for four transmitted antennas over uncorrelated and spatially correlated MIMO channels is investigated. A maximum likelihood (ML) decoding algorithm for the suggested code is also provided.**

***Index Terms*—Diversity, space time codes, MIMO systems, quasi-orthogonal codes.**

I. Introduction

A space–time code (STC) is a method employed to improve the reliability of data transmission in wireless communication systems using multiple transmit antennas [1], [2]. STCs rely on transmitting multiple, redundant copies of a data stream to the receiver in the hope that at least some of them may survive the physical path between transmission and reception in a good enough state to allow reliable decoding. The Alamouti code [1] is the first STBC that provides full diversity at full data rate for two transmit antennas. However, if the transmitting antennas are more than two, using orthogonal space time codes would not obtain the maximum transmission rate [3], [4]. So Jafarkhani proposed a space time code structure based on the full rate [5]. In Jafarkhani's method, the transmission matrix columns are divided into groups .while the columns within each group are not orthogonal to each other, different groups are orthogonal to each other. Thus this method is also called the quasi-orthogonal space time block codes, another code was proposed in [6] called TBH (ABBA) it give the same performance as Jafarkhani when there is no correlation in channel but for spatially correlated channel the performance of this codes degrade compared to Jafarkhani. The suggested codes are designed by replacing the Alamouti based sub block matrix with different matrices as in Jafarkhani [5], and TBH [6].

II. System Model

Let the number of transmitting antennas and receiving antennas is N and M respectively, a complex space time block codes is given by $T\times N$ transmission matrix G, T represents the number of time slots for transmitting one block of symbols, supposing quasi-static flat rayleigh fading channel the receiver vector will be $r(k)=H(k)s(k)+n(k)$ where $r(k)$ is the receiving vector , $s(k)$ is the transmitting vector and n(k) is additive white Gaussian noise and H is the channel matrix and α_{ij} is the fading coefficients from the i transmitting antenna to j transmitting antennas.

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III. Space Time Block Codes

A. Alamouti Space Time Code

Historically, the Alamouti code is the first STBC that provides full diversity at full data rate for two transmit antennas [1]. The information bits are first modulated using an M-ary modulation scheme. The encoder then takes a block of two modulated symbols s_1 and s_2 in each encoding operation and gives it to the transmit antennas according to the code matrix,

$$s=\begin{bmatrix} s_1 & s_2 \\ -s_2^* & s_1^* \end{bmatrix} \tag{1}$$

In equation (1), the first row represents the first transmission period, whereas the second row the second represents the transmission period. The first and the second columns correspond to the symbols transmitted from the first and the second antenna respectively.

B. Jafarkhani Quasi Orthogonal Codes

Based on Alamouti scheme, Jafarkhani construct his quasi orthogonal space time block codes for four antennas. Two (2 × 2) Alamouti codes S12 and S34 are defined equation (2), and they are used as sub-blocks to build Jafarkhani code for four transmit antennas.

$$S_{12}=\begin{bmatrix} s_1 & s_2 \\ -s_2^* & s_1^* \end{bmatrix} \qquad S_{34}=\begin{bmatrix} s_3 & s_4 \\ -s_4^* & s_3^* \end{bmatrix}$$

Using these matrices as sub-block matrix, the coding matrix of Jafarkhani codes could be expressed as follows.

$$S_{EA}=\begin{bmatrix} S_{12} & S_{34} \\ -S_{34}^* & S_{12}^* \end{bmatrix}=\begin{bmatrix} s_1 & s_2 & s_3 & s_4 \\ -s_2^* & s_1^* & -s_4^* & s_3^* \\ -s_3^* & -s_4^* & s_1^* & s_2^* \\ s_4 & -s_3 & -s_2 & s_1 \end{bmatrix} \tag{2}$$

The decoder is based on the multiplication of S_{EA} with its Hermitian matrix leading to the non-orthogonal Gramian matrix Q_{EA}. A Gramian matrix A is a Hermitian symmetric matrix that fulfils $A^H = A$, where H indicates conjugate-transpose [7].

$$Q_{EA} = S_{EA}^H S_{EA} = S_{EA} S_{EA}^H$$
$$= s^2 \begin{bmatrix} 1 & 0 & 0 & \gamma_{EA} \\ 0 & 1 & -\gamma_{EA} & 0 \\ 0 & -\gamma_{EA} & 1 & 0 \\ \gamma_{EA} & 0 & 0 & 1 \end{bmatrix}$$

where $s^2 = |s_1|^2 + |s_2|^2 + |s_3|^2 + |s_4|^2$

$$\gamma_{EA} = \frac{2\,\mathrm{Re}(s_1 s_4^* - s_2 s_3^*)}{s^2} \quad (3)$$

γ_{EA} is the channel dependent interference parameter, it is the interference parameter that define how close such a matrix to be orthogonal From equation (3), the symbols s_1, s_4 and the symbols s_2, s_3 appear in pairs, a fact that simplifies the analysis of the code.

C. TBH Quasi orthogonal Codes

Two (2 × 2) Alamouti codes S_{12} and S_{34}, as shown in equation (2), are used as sub-blocks to build the TBH code, also called the ABBA code, [6] for four transmit antennas.

$$S_{ABBA} = \begin{bmatrix} S_{12} & S_{34} \\ S_{34} & S_{12} \end{bmatrix} = \begin{bmatrix} s_1 & s_2 & s_3 & s_4 \\ -s_2^* & s_1^* & -s_4^* & s_3^* \\ s_3 & s_4 & s_1 & s_2 \\ -s_4^* & s_3^* & -s_2^* & s_1^* \end{bmatrix} \quad (4)$$

The multiplication of the code matrix S_{ABBA} by its Hermitian yields to the following non-orthogonal Gramian matrix Q_{ABBA}.

$$Q_{ABBA} = S_{ABBA}^H S_{ABBA} = S_{EA} S_{EA}^H = s^2 \begin{bmatrix} 1 & 0 & \gamma_{ABBA} & 0 \\ 0 & 1 & 0 & \gamma_{ABBA} \\ \gamma_{ABBA} & 0 & 1 & 0 \\ 0 & \gamma_{ABBA} & 0 & 1 \end{bmatrix}$$

where

$$\gamma_{ABBA} = \frac{2\,\mathrm{Re}(s_1 s_3^* + s_2 s_4^*)}{s^2}. \quad (5)$$

IV. Construction of Suggested Codes

A. Suggested Code 1

According to the above analysis by changing the Alamouti based matrix S_{12} a new transmitting coding matrices could be obtained based on the position of the correlated values in the Gramian matrix [7],[8].Let the sub-blocks matrices be as follows.

$$S_{12} = \begin{bmatrix} s_1 & s_2 \\ s_2 & s_1 \end{bmatrix} \qquad S_{34} = \begin{bmatrix} s_3 & s_4 \\ s_4 & s_3 \end{bmatrix} \quad (6)$$

In this case we did not take the Alamouti as the sub-block matrix; the new design is expressed in equation (7).

$$C_{new1} = \begin{bmatrix} S_{12} & -S_{34} \\ S_{34}^* & S_{12}^* \end{bmatrix} = \begin{bmatrix} s_1 & s_2 & -s_3 & -s_4 \\ s_2 & s_1 & -s_4 & -s_3 \\ s_3^* & s_4^* & s_1^* & s_2^* \\ s_4^* & s_3^* & s_2^* & s_1^* \end{bmatrix} \quad (7)$$

By using equation (6) as a sub-block, 16 coding matrices can be generated using the four matrices that are defined in equation (8) and using their negative and / or conjugated versions (i.e. $-C$, C^*, and $-C^*$).

$$\begin{bmatrix} S_{12} & S_{34} \\ S_{34}^* & -S_{12}^* \end{bmatrix}, \begin{bmatrix} S_{12} & S_{34} \\ -S_{34}^* & S_{12}^* \end{bmatrix}, \begin{bmatrix} S_{12} & -S_{34} \\ S_{34}^* & S_{12}^* \end{bmatrix}, \begin{bmatrix} -S_{12} & S_{34} \\ S_{34}^* & S_{12}^* \end{bmatrix} \quad (8)$$

By multiplication of the code matrix C_{new1} by its Hermitian the following non-orthogonal Gramian matrix Q_{new1} is obtained:

$$Q_{new1} = S_{new1}^H S_{new1} = s^2 \begin{bmatrix} 1 & \gamma_{new1} & 0 & 0 \\ \gamma_{new1} & 1 & 0 & 0 \\ 0 & 0 & 1 & \gamma_{new1} \\ 0 & 0 & \gamma_{new1} & 1 \end{bmatrix}$$

where

$$\gamma_{new1} = \frac{2\,\mathrm{Re}(s_1 s_2^* + s_3 s_4^*)}{s^2} \quad (9)$$

For decoding this code, the receiver computes the decision metric that is defined in equation (10) over all possible $x_k = s_k$ and making a decision in favour of the constellation symbols that minimize this sum. Perfect channel state information is assumed.

$$\sum_{m=1}^{M} \sum_{t=1}^{T} \left| r_{t;m} - \sum_{n=1}^{N} \alpha_{n,m} G_{tn} \right|^2 \quad (10)$$

The maximum likelihood decision metric, in equation (10), can be calculated as the sum of $f_{12}(s_1,s_2)+f_{34}(s_3,s_4)$ where f_{12} is independent from s_3, s_4 thus minimization of equation (10) is equivalent to the minimization of each two symbols separately. This reduces the complexity of the decoder, since instead of decoding four symbols all together it decodes each two separately.

Simple manipulation of equation (10) yields to the following formulas. Equation (11) is used to decode s_1, s_2, whereas equation (12) is used for decoding s_3, s_4.

$$f_{12}(s_1,s_2)=\left(\sum_{n=1}^{4}\left(|\alpha_n|^2\right)\right)\left(|s_1|^2+|s_2|^2\right)+2\,\mathrm{Re}\begin{Bmatrix}(-r_1^*\alpha_1-r_2^*\alpha_2-r_3\alpha_3^*-r_4\alpha_4^*)x_1\\+(-r_1^*\alpha_2-r_2^*\alpha_1-r_3\alpha_4^*-r_4\alpha_3^*)x_2\\+(\alpha_1\alpha_2^*+\alpha_1^*\alpha_2+\alpha_3^*\alpha_4+\alpha_3\alpha_4^*)x_1x_2^*\end{Bmatrix} \quad (11)$$

$$f_{34}(s_3,s_4)=\left(\sum_{n=1}^{4}\left(|\alpha_n|^2\right)\right)\left(|s_3|^2+|s_4|^2\right)+2\,\mathrm{Re}\begin{Bmatrix}(r_1^*\alpha_3+r_2^*\alpha_4-r_3\alpha_1^*-r_4\alpha_2^*)x_3\\+(r_1^*\alpha_4+r_2^*\alpha_3-r_3\alpha_2^*-r_4\alpha_1^*)x_4\\+(\alpha_3\alpha_4^*+\alpha_3^*\alpha_4+\alpha_1^*\alpha_2+\alpha_1\alpha_2^*)x_3x_4^*\end{Bmatrix} \quad (12)$$

B. Suggested Code 2

The sub-block matrices S_{12}, S_{34} are defined as follows:

$$S_{12}=\begin{bmatrix}s_1 & -s_2\\ s_2 & s_1\end{bmatrix} \qquad S_{34}=\begin{bmatrix}s_3 & -s_4\\ s_4 & s_3\end{bmatrix}$$

Thus the suggested design is as defined in equation (13).

$$C_{new2}=\begin{bmatrix}S_{12} & -S_{34}\\ S_{34}^H & S_{12}^H\end{bmatrix}=\begin{bmatrix}s_1 & -s_2 & -s_3 & s_4\\ s_2 & s_1 & -s_4 & -s_3\\ s_3^* & s_4^* & s_1^* & s_2^*\\ -s_4^* & s_3^* & -s_2^* & s_1^*\end{bmatrix} \quad (13)$$

The multiplication of the code matrix C_{new2} by its Hermitian yields to the following non-orthogonal Gramian matrix Q_{new2}.

$$Q_{new2}=S_{new2}^H S_{new2}=s^2\begin{bmatrix}1 & \gamma_{new2} & 0 & 0\\ -\gamma_{new2} & 1 & 0 & 0\\ 0 & 0 & 1 & \gamma_{new2}\\ 0 & 0 & -\gamma_{new2} & 1\end{bmatrix}$$

where $\gamma_{new2}=\dfrac{2\,\mathrm{Im}(s_1s_2^*+s_3s_4^*)}{s^2}$ is an imaginary number.

And for ML decoding, we use the following equations to decode each pair separately

$$f_{12}(s_1,s_2)=\left(\sum_{n=1}^{4}\left(|\alpha_n|^2\right)\right)\left(|s_1|^2+|s_2|^2\right)+2\,\mathrm{Re}\begin{Bmatrix}(-r_1^*\alpha_1-r_2^*\alpha_2-r_3\alpha_3^*-r_4\alpha_4^*)s_1\\+(-r_1^*\alpha_2+r_2^*\alpha_1+r_3\alpha_4^*-r_4\alpha_3^*)s_2\\+(\alpha_1\alpha_2^*-\alpha_1^*\alpha_2-\alpha_3^*\alpha_4+\alpha_3\alpha_4^*)s_1s_2^*\end{Bmatrix}$$

$$f_{34}(s_3,s_4)=\left(\sum_{n=1}^{4}\left(|\alpha_n|^2\right)\right)\left(|s_3|^2+|s_4|^2\right)+2\,\mathrm{Re}\begin{Bmatrix}(r_1^*\alpha_3+r_2^*\alpha_4-r_3\alpha_1^*-r_4\alpha_2^*)s_3\\+(r_1^*\alpha_4-r_2^*\alpha_3+r_3\alpha_2^*-r_4\alpha_1^*)s_4\\+(\alpha_3\alpha_4^*-\alpha_3^*\alpha_4-\alpha_1^*\alpha_2+\alpha_1\alpha_2^*)s_3s_4^*\end{Bmatrix}$$

V. Simulation Results

In this section the simulation results that demonstrate the efficiency of the suggested codes are demonstrated. First it is assumed that Rayleigh fading channel is kept constant during the transmission of each code block but it changes independently between successive blocks, the receiver has perfect CSI, and the fading between transmit and receive antennas is mutually independent. For the suggested codes QPSK modulation is used at the transmitter, with constellation rotation with angle that equals π/4 for s_3, s_4 [9] to achieve full diversity. The bit error rate versus signal to noise ratio is shown in Fig. 1 for Jafarkhani codes, TBH codes, and the two suggested codes. From Fig. 1 the two new codes have better BER than Jafarkhani and TBH.

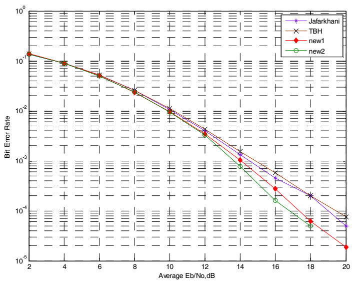

Fig. 1 The bit error rate performance comparison for QSTBCs

Fig. 2 shows a comparison between 16 different codes using the four coding matrices in equation (8) with their negative, conjugate and negative conjugate. Fig. 2 shows that the bit error rate for the 16 codes is the same because they have the same value for correlation coefficient, so any of these coding matrices code is used to represent the suggested code 1.

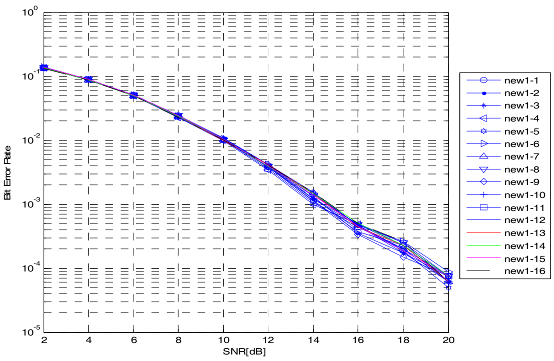

Fig. 2. Using 16 different matrices to represent new1 code

Each code was simulated on i.i.d. (independent and identically-distributed) channels and on spatially correlated channels [8] (high correlation, $\rho = 0.95$) using the following correlation matrix:

$$\begin{bmatrix} 1 & \rho & \rho^2 & \rho^3 \\ \rho & 1 & \rho & \rho^2 \\ \rho^2 & \rho & 1 & \rho \\ \rho^3 & \rho^2 & \rho & 1 \end{bmatrix} \tag{14}$$

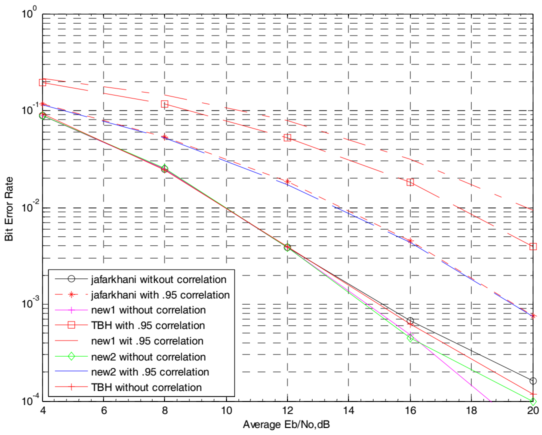

Fig. 3. Comparison of code designs on spatially uncorrelated and spatially correlated MIMO channels (ρ= 0.95) with ML receiver.

Fig. 3 shows comparison between suggested codes and already known codes in spatially uncorrelated channel and spatially correlated channel. The difference between the code designs in fig.3 is the self-interference parameter showing up in the corresponding non-orthogonal Gramian matrices. In fact, the allocation of interference parameter in the corresponding Gramian matrix does not affect the code performance. This is confirmed by their identical performance in spatially uncorrelated channels. However, in spatially correlated channels a substantial performance difference is observed due to different values of the self-interference parameter.

VI. Conclusions

In this paper, two suggested codes are demonstrated for Quasi orthogonal space time codes. Different codes could be obtained by changing sub-block matrix to obtain various values for the self interference parameter. The performance of these codes compared to Jafarkhani and TBH, the simulation results have demonstrated that performance of these codes differ in spatially correlated channel depending on the value of self interference parameter γ .

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