Signal Reconstruction by Using the Samples of Derivative Signal in Asynchronous Level-Crossing Converter

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Abstract: This paper presents a new method which could reconstruct a signal by using the samples of derivative signal. Using this method in Asynchronous Level Crossing converters which introduced non-uniform samples, leads to a significant reduction in number of samples which are needed to reconstruct a signal. This method is beneficial for signals which could be estimated as a polynomial function in some part of its domain. The time instances of each sample should be sent too.

Key words: Analog to digital converter, asynchronous level crossing converter, non-uniform sampling, signal reconstruction.

1. Introduction

This paper proposes a method to reconstruct a signal with samples of derivative signal with Level Crossing converters. LC converter is asynchronous in nature and provides interesting properties such as low power dissipation, electromagnetic interference reduction and improved Figure-of–Merit compared to the conventional A/D's [1]. In addition the LC converters can match themselves to the varying dynamic range of the input analog signal without any loss of quality while conventional A/D's create more distortion if the dynamic range suddenly changes. Also for remote sensing signals that are almost constant and may vary just during limited moments, asynchronous A/Ds decrease the activity of the circuit in comparison with synchronous A/Ds and therefore lower power consumption [1]. For non-band-limited signals; Nyquist sampling cannot capture the characteristics of the input.

LC converters can be more useful in these situations [2]. The architecture of Asynchronous A/Ds based on level crossing scheme has been proposed in [1]. A level-crossing scheme for A/Ds has been discussed in [1] where the non-uniform sampling sequence is transformed to uniform samples by polynomial interpolation. The LC scheme of non-band limited signal has been proposed in [2] and compared to uniform sampling the authors have shown that this type of sampling outperforms uniform sampling.

This paper shows that by calculating the linear derivation and use the sample of derivative signal, we could reconstruct the original signal properly. Using the derivative signal fortransmitting, instead the original signal in LC converter, yields to reduce the activity of LC converter. Any reduction in number ofsamples which transmit the information of the signal with acceptable quality, is desired. Section 2 gives the principles of uniform and non-uniform sampling and synchronous and asynchronous converters. Also the level crossing converters scheme is given in this method is described and in Section 4 the simulation results are shown. Section 5 concludes this work.

2. Basic Principles

2.1. Sampling

2.1.1. Uniform sampling

The Sampling theorem states: each signal function *f* that is band-limited to [-*w*, *w*] for some *w*>*0* , i.e., *f* is square integrable (finite energy) and contains no frequencies higher than *w*, thus

$$f(t) = \frac{1}{2\pi}\int_{-w}^{w} F(u)e^{jut}du \qquad (t\epsilon\mathbb{R}) \tag{1}$$

Can be completely reconstructed from its sampled values $f\left(\frac{k\pi}{w}\right)$ taken at the instants $\frac{k\pi}{w}$, $k\epsilon\mathbb{Z}$, equally spaced apart on $\mathbb{R}$ in terms of

$$f(t) = \sum_{-\infty}^{\infty} f\left(\frac{k\pi}{w}\right)\mathrm{Sinc}\left(\frac{wt}{\pi} - k\right)\ (t\epsilon\mathbb{R}) \tag{2}$$

The series $S_w f$ converges absolutely and uniformly forany $t\epsilon\mathbb{R}$.

In addition we could reconstruct any order of derivative signal by use this samples.

$$f^{(r)}(\mathrm{t})=\sum_{k=-\infty}^{\infty} f\left(\frac{k\pi}{w}\right)\left(\frac{d}{dt}\right)^{r}\mathrm{sinc}\left(\frac{wt}{\pi} - k\right) \tag{3}$$

For example

$$f'(t) = w\sum_{k=-\infty}^{\infty} f\left(\frac{k\pi}{w}\right)\left\{\frac{\cos(wt-k\pi)}{wt-k\pi} - \frac{\sin(wt-k\pi)}{(wt-k\pi)^2}\right\} \tag{4}$$

Two conditions must to be considered for sampling.

1) The signal has finite energy.

2) The signal has finite extermums.

Furthermore, the sampling frequency must be at least twice the signal bandwidth which is sampled. In Fig. 1 the effect of sampling rate is shown. If sampling rate is less than this, we lose some of signal information and we couldn't reconstruct the signal properly.

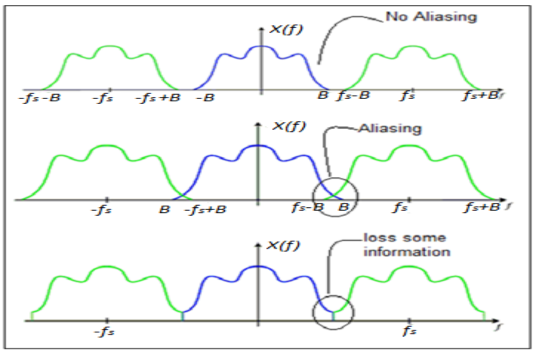


Fig. 1. The effect of sampling frequency.

This frequency is called Nyquiest frequency (f_N) [1] which is shown in (5).

$$fs_{(\min)} = f_N = BW \times 2 \tag{5}$$

Quantization process incurs loss of information in reconstructed signal. In order To overcome this problem the rate of sampling should be greater than Nyquist rate. The ratio between the sampling frequency and f_N is called Over Sampling Ratio (OSR) [3] which is the main condition that could influence the quality of reconstructed signal in many converters.

$$OSR = {f_S}/{f_N} \tag{6}$$

2.1.2. Non-uniform sampling

Shannon in 1949 in [4] indicated that a band limited signal can be reconstructed from the values of the signal at non-uniformly spaced points and also from the values of the signal and its derivative. He said $2TW$ number, that is not necessary be uniformly spaced, are adequate to describe it. He then proceeded to say: "One can further show that the value of the function and its derivative at every other sample point are sufficient (to specify the function).

If we do non-uniform sampling operation by considering the Nyquist rate, we could reconstruct the signal by using these non-uniform samples [4]. This means that the average number of sample be greater than Nyquist rate, the signal reconstruction is possible. Also if the samples values and times instances of these samples be sent, it is possible to reconstruct the signal in sub Nyquist rate [3].

Non-uniform sampling conception in electronic and communication is tied with asynchronous converters.

There are some reconstructing methods for non-uniform sampling. Lagrange interpolation method [3] is one of them. The almost all the known interpolation formulas for the non-uniform sampling reconstruction can be derived from the Lagrange interpolation [3].

Also reconstruction with non-uniform samples can be done by $TimeVariation$ method, iterative method or $MatrixApproximation$ method [3], [5].

2.2. Synchronous and Asynchronous Converters

In synchronous converters, uniform samples are quantized to approximate a continuous range of values by a specific set of discrete values.

Quantization process incurs loss of information in reconstructed signal. In order to overcome this problem OSR should be greater than Nyquist rate ($f_s >> f_N$).

In asynchronous converters the sampling is done irregularly. In non-uniform sampling the mean value of sampling rate should be greater than (f_N). Nonuniform sampling could be done with level crossing converters. In this type of converters [6]-[8], the difference time between two samples are quantized and transmitted [6], [9].

2.3. Partial and Linear Derivation

The mathematical interpretation for deriving a signal [10] is indicated in (7).

$$f'(t) = \lim_{\Delta t \to 0}(f(t + \Delta t) - f(t))/\Delta t \tag{7}$$

If Δt is not near to zero but be enough little, in each time we have an approximated value of partial derivation. This approximation is calculated by using the values of two points of the function. By this we achieve the [8], [10].

$$f'(t_n) = \Delta f_n/\Delta t_n = (f(t_n) - f(t_{n-1}))/(t_n - t_{n-1}) \quad (8)$$

Whatever these samples of function f are closer to each other, this linear derivation is more accurate and be a better estimation of the partial derivation. It means that if we have greater sampling rate, the linear derivation is more accurate and is a better approximation of partial derivation.

2.4. Asynchronous Level Crossing Converter

For a non-uniform sampling A/D converter, the conversion of samples takes place whenever a reference level is crossed by the continuous time signal. Hence, the amplitude of the sample is precise, but we need some kind of quantization in the time domain. This type of sampling is called Level Crossing (LC) [7],[8]. When a reference level is crossed by a continuous time signal, the precise value of level amplitude a_0, and the time difference between two level crossings dt_0 should be transmitted. For digital transmission, dt_n must be quantized. To prevent the quantization error propagation, it is better to calculate time difference due to the quantized time of previous sample $Q\ (t_{n-1})$ instead of $t_{n-1}.Q(t_{n-1})$ is already known from the quantized values of t_{n-1}

$$dt_n = t_n - Q(t_{n-1}) \quad (9)$$

The Effective Number of Bits (ENOB) is defined as the number of bits allocated to the quantized dt_n.Fig. 5 shows the sampling in level crossing converter.

Whatever the distances between the levels are smaller, the sampling is done with more resolution and consequently the reconstructed signal is a better estimation of the original signal. Also the clock signal frequency which determines the resolution of sampled signal. The inverse portion of time resolution directly influences the signal to noise ratio.

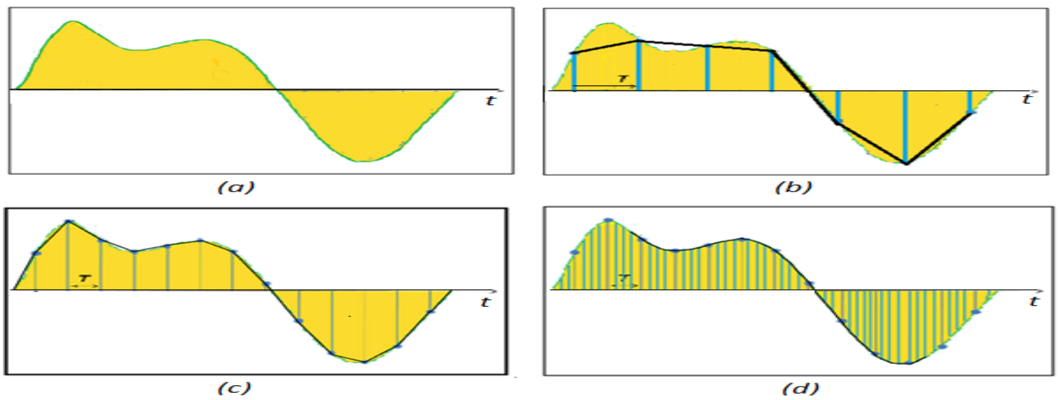


Fig. 2. The effect of OSR on the reconstructed signal. (a) the original signal. (b) sampling with a low frequency and reconstructed signal that have a considerable error. (c) and (d) use higher OSR and achieve the better signal and lower error.

3. Proposed Method

In this new method we could reconstruct the signal by using the samples of linear derivation. Linear derivation is calculated by using two consecutive of uniform samples. It means that at first we should do a sampling process on the signal. Then a processor uses these samples and calculates the linear derivation with (7). If the sampling frequency is enough large this estimation is more accurate and be closer to partial derivation in (8). The inputs and outputs of this block are digital or in the other word, they are discrete time values. So, this block is a digital processor.

To reconstruct the signal with the samples of derivative signal, signal must have two conditions.

1) Signal should be continuous and don't have abrupt variations.
2) Signal should be band limited.

If any of these two conditions aren't observed, in order to achieve these conditions, we can use a low pass filter to obtain a good signal. Fig. 3(a) shows a square form signal with abrupt variations. Fig. 3(b) shows the filtered signal that its variations have been smoothed.

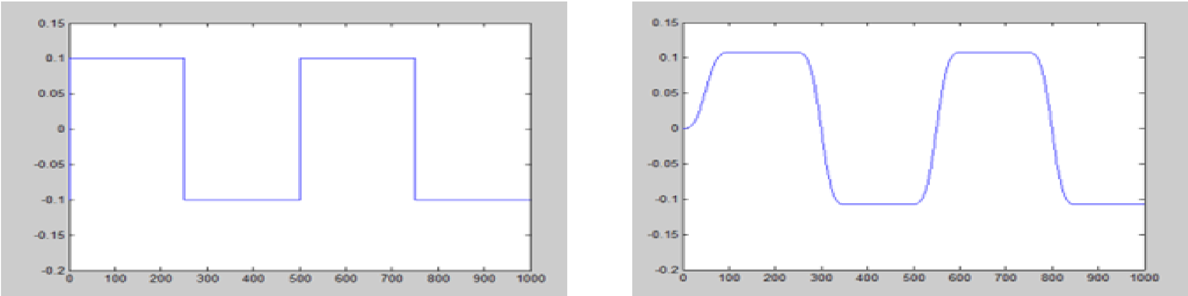


Fig. 3. (a) Square wave signal, (b) filtered signal.

Equation (10) has been used in the derivator. To reconstruct the signal we must have the value of signal at time t=0. Then by using the (11) we could reconstruct the signal properly.

$$f'(t_n) = f(t_n) - f(t_{n-1}) \tag{10}$$

$$f(t_{n+1}) = \mathrm{f}(t_n) + \mathrm{f}'(t_n) \tag{11}$$

When we use level crossing converter, the number of samples is dependent to the domain of signal variations and rate of variations. More samples are taken to transmit, if the variations domain is large or if the signal has high rate variations. But if the variations domain is small or variations have a low rate, the number of samples is less. The power consumption of this converter is dependent to the number of samples which should to be transmitted. Then it could be an aim to send a signal with fewer samples. Fig. 4 shows the sampling in a LC converter.

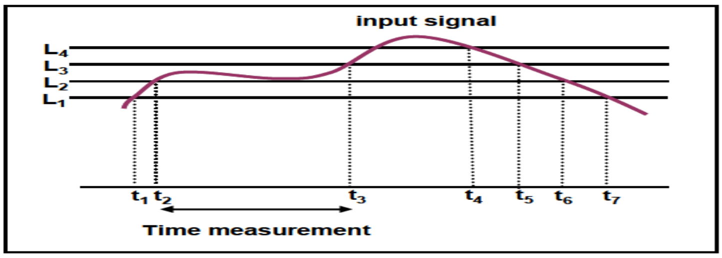


Fig. 4. Sampling in level crossing converter.

The main benefit of proposed method is reduction of needed number of samples to reconstruct a signal. The signal should has a main property: If a signal could be separated to some variations areas and it's possible to estimate one or some of this area with the polynomial.

$$x(t) = \ a_n t^n + a_{n-1} t^{n-1} + \ldots + a_1 t + a_0 \tag{12}$$

$$x^{'}(t) = n a_n t^{n-1} + (n - 11) a_{n-1} t^{n-2} + \cdots + a_1 \tag{13}$$

$$x^n(t) = n(n-1)(n-2)\ldots.(2)(1)a_n \tag{14}$$

To reconstruct a signal with first order derivative signal the value of the signal in t=0 is needed. If we want to recover the signal with second order derivative signal the value of signal in t=0 and t=1 are needed.

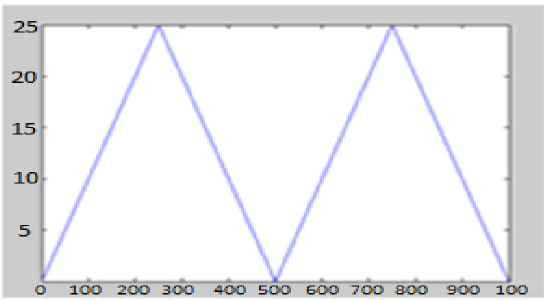
Therefore by calculating the derivative signal and done the sampling process with LC converter, could reduce the number of samples which should to be transmitted to reconstruct a signal.

4. Simulation Results

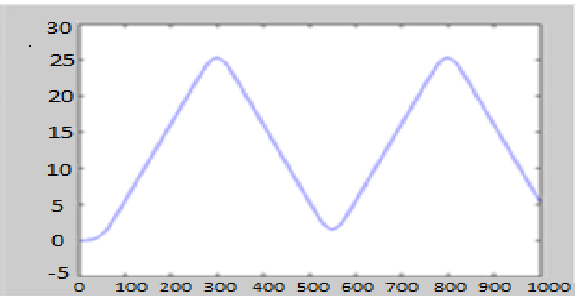
In Fig. 2 we could see the effect of TR on the reconstructed signal and error. At Fig. 5 the simulation results of a triangle signal is shown. A triangular wave was selected as original signal. The derived signal of this wave is a square wave and has abrupt variations. These abrupt variations could make some error in the reconstructed signal. To overcome this limitation we used a low pass FIR filter to smoothing the sudden variations. The filtered signal is shown in Fig. 5(b). This figure shows that sudden variations which are in peaks of the triangle signal are smoothed. The derived signal of filtered signal is shown in Fig. 5(c). The reconstructed signal by proposed method is shown in Fig. 5(d). And the obtained SNR is 80db.

If the signal is reconstructed by the samples of original signal, the distance between the levels in level crossing converter should be ten times greater than distances between the levels of level crossing converter in proposed method to have SNR=40db.

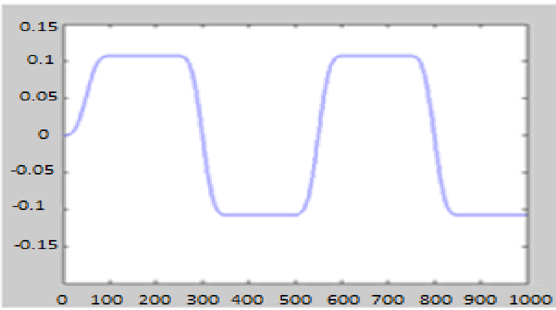
Using this method yields to a considerable reduction of the samples which should to be transmitted. This decrement reduces the power consumption for transmitting a signal. The simulation results show that if we want to send a filtered triangle signal without this method and by level crossing converter, 1250 samples have been needed for SNR=40 db. But by this proposed method, 345 samples are sufficient to have SNR=40 db.



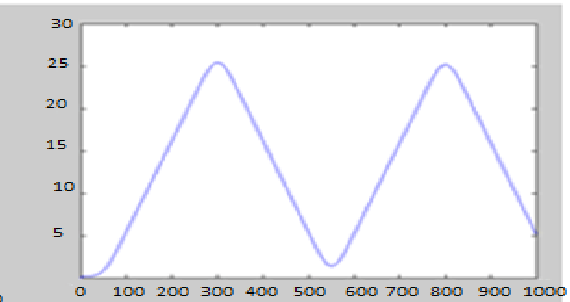
(a)



(b)



(c)



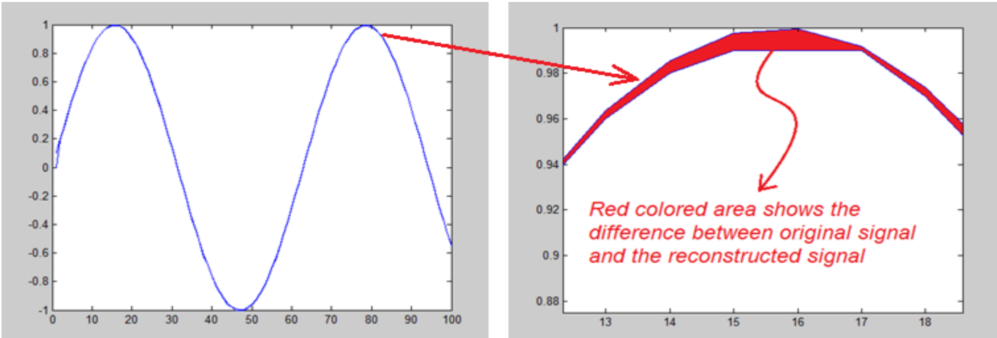
(d)

Fig. 5. (a) The original signal; (b) filtered signal; (c) derived signal; (d) reconstructed signal.

Fig. 6 shows a sin wave signal and its reconstructed which was sampled with LC.

Fig. 7 shows the effect of Time Resolution (TR) or in the other word the frequency of clock signal on reconstructed signal. Whatever the sampling TR is smaller the recovered signal is a better approximation of the original signal. Fig. 7 shows the difference between the recovered signal and original signal with various values of TR.

Fig. 8 shows the effect of Time Resolution on the obtained Signal to Noise Ratio (SNR).



(a) (b)

Fig. 6. (a) Both of the original signal and reconstructed signal are shown. (b) zoom in the maximum area and the red colored area shows the difference between the original signal and reconstructed signal.

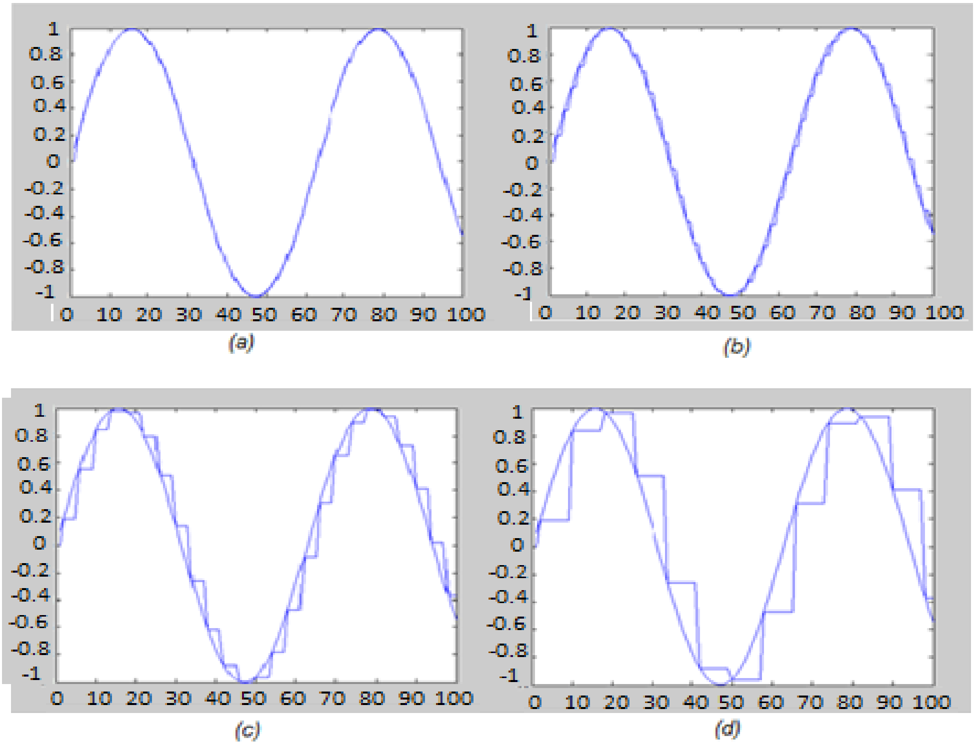


Fig. 7. Effect of TR on the reconstructed signal. (a) TR= 1ms, (b) TR= 2ms, (c) TR= 4ms, (d) TR= 8ms.

5. Conclusion

In this paper a new method to reconstruct a signal by using the samples of derivative signal has been presented. This method used the level crossing converter. In this type of converter we deal with non-uniform sampling conception. Using the proposed method leads to send and reconstruct a signal with a significant fewer number of samples. Proposed method is for LC converter and is beneficial for signals which could be separated to some variation areas and If it's possible to estimate one or some of these areas with a polynomial. Also the resulted SNR is proportional to the minimum time resolution.

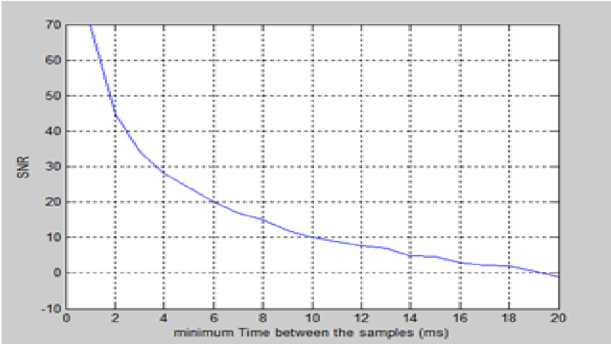


Fig. 8. The effect of time resolution on resulted SNR.

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