Design of a Merged Algorithm for Luby Transform Decoder

Hanady Hussien, Khaled A. Shehata, Salwa El Ramly, and Nihal M. S. Tawfik

***Abstract*—Luby transform exhibits near-optimal performance over Binary Erasure channel. However, on AWGN channel, Luby decoding technique suffers from error propagation. Consequently, a soft decoding strategy -Belief Propagation- similar to the LDPC has been adopted. In this strategy, the check node equation complexity is still a persistent problem affecting hardware implementation in terms of speed and area. We propose a decoding scheme that uses both Luby decoding technique and the soft input available at the receiver to reduce the check node equation complexity. In the proposed algorithm, error propagation has been mitigated thus reducing the signal-to-noise ratio significantly.**

***Index Terms*—Belief propagation, hard decision decoding, Luby transform codes, soft decision decoding, sum product algorithm**

I. Introduction

Automatic Repeat reQuest (ARQ) is a common successful method for providing reliability over unicast IP-based networks. For IP-multicast, ARQ can be highly inefficient especially when many receivers require the simultaneous retransmission of different packets. This was the main motivation behind the invention of Fountain Codes.

Fountain codes were invented for packet based multicast scenarios over wired Internet in which packets are either received or lost. This behaviour is modelled by a Binary Erasure Channel (BEC).

These codes make use of a simple idea: the original data can be entirely recovered from any set of received bits under condition that the received bits are slightly larger than the original. Since no fixed rate is assigned, fountain codes are also referred to as rateless codes.

The first practical implementation of Fountain codes is Luby Transform (LT) codes [1]. Following them, Raptor Codes were introduced [2], where the concatenation of Luby Transform with a simple block code as a pre-code is used to increase the performance.

LT codes simple hard-decision decoding strategy has high performance over BEC. However, over noisy channel this hard-decision technique encounters high error propagation. In the last few years, a superior decoding strategy that uses the sum product algorithm (SPA), similar to the low density parity check codes , has been developed to investigate the performance of LT codes over noisy channels [3–5]. This decoding strategy exploits the soft input available at the decoder. For speed optimization purpose, LT hard-decision decoder was first implemented on FPGA [6]. An applicable architecture of a soft decision decoder prototyped on FPGA was also presented in [7]. The check node complexity in SPA has always hampered efficient implementation. Researches in LDPC have made many modifications to simplify the SPA. A new algorithm to merge between hard-decision decoding and soft decision decoding in LDPC to achieve a trade-off between the low performance of the first and the implementation complexity of the second have recently been developed by [8]. This paper introduces a similar strategy developed for Luby Transform codes. The rest of the paper is organized as follows: Section II presents LT hard and soft decision decoding techniques. Section III introduces the new developed decoding algorithm. Section IV discusses the simulation results of the new technique followed by the conclusion on Section V.

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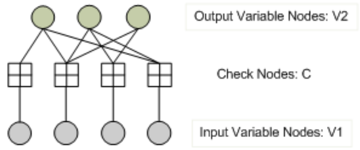
II. LT Codes

A. LT Encoder

An LT encoder generates the encoded data packets from a K number of input message packets m_1, m_2, m_3,...m_K as follows [9]:

1) From a degree distribution function *ρ(d)*, select a degree d_n. The degree distribution function is designed according to the size K of the source file to be encoded.
2) Select a number d_n of input packets to be encoded, the encoded packet c_n is the xoring of those packets together.

Fig. 1 shows the Tanner graph of the generator matrix G resulting from the encoding process which consists of two variable nodes denoted in this paper as variable node V1 and variable node $V2$ and one check node C.



$$G = \begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix}$$

Fig. 1. LT tanner graph.

B. Degree Distribution

Degree Distribution is a crucial part in the encoding process. Many packets must have a low degree so that the decoding process can start and some other packets must have a high degree to ensure that no packets are left unconnected.

Luby [1] has first introduced Robust Soliton Distribution, other distribution have been proposed in [2], [10]. In this paper two main distributions are compared, Robust Soliton distribution and a recently proposed distribution [11] optimized for rate 1/2 shown in the equation:

$$\rho(d) = 0.1403x + 0.4920x^2 + 0.1472x^3 + 0.0905x^4 + 0.0739x^5 + 0.0060x^8 + 0.0110x^{14} + 0.0346x^{30} \quad (1)$$

where the power of x represents d_n.

C. LT Hard Decision Decoder

First we have to find a check node that is connected to only one source packet.

- Set $m_k = c_n$.
- Xor m_k to all check nodes c_n that are connected to m_k.
- Repeat the above steps until all input symbols are recovered.
- Remove all edges connected to the source packet m_k .

The process is then repeated until all connections are removed. The decoding algorithm, illustrated with a simple example [9] shown in Fig. 2, corresponds to solving the set of linear equations shown.

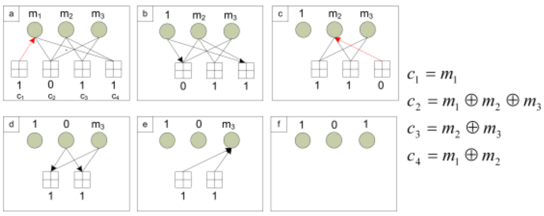


Fig. 2. LT decoding process.

It is clear from the above that only one corrupted packet will lead to major degradation in the decoding process as depicted in [12].

D. LT Soft Decision Decoder

The sum-product algorithm for Belief-Propagation (BP) decoding is applied to the tanner graph show in Fig. 1. The probability domain SPA involves many multiplications of probabilities which lead to numerical instability and high hardware implementation complexity. Therefore, the log-domain SPA is preferable; it is described below.

The Log-Likelihood Ratios (LLR) of the received values are first computed at $V1$:

$$L(c_n) = \log\left(\frac{\Pr(c_n = +1)}{\Pr(c_n = -1)}\right) \quad (2)$$

Algorithm:

Define: j,n = 1,2,….Degree of the check node C
i,k = 1,2,….Degree of the variable node $V2$

1) Compute the message from the check node C to the variable node V2:

$$L(t_{n\to k}) = \alpha_{L(c_n)} \prod_{i\neq k} \alpha_{L(h_{i\to n})} \cdot \varphi\left[\varphi\left(\beta\left(L(c_n)\right)\right) + \sum_{i\neq k} \varphi\left(\beta\left(L(h_{i\to n})\right)\right)\right] \quad (3)$$

where:

$$\alpha_x = sign(x)$$
$$\beta_x = |x|$$
$$\varphi(x) = \log\left(\frac{e^x + 1}{e^x - 1}\right)$$

2) Compute the message from the variable node $V2$ to the check node C:

$$L(h_{k\to n}) = \sum_{j\neq n} L(t_{k\to j}) \quad (4)$$

3) The L-value of the decoding message is given by:

$$L(m_k) = \sum_{j=n} L(t_{k\to j}) \quad (5)$$

4) Decision is made at $V2$ after a certain number of iterations:

$$m_k = \begin{cases} 1, & if\ L(m_k) < 0 \\ 0, & if\ L(m_k) > 0 \end{cases} \quad (6)$$

All messages are initially set to zero except for $L(c_n)$ received at the variable node $V1$.

III. Proposed Algorithm

As clarified in the above sections, LT decoding suffers either from the inevitable error propagation of its hard-based algorithm or from the check node complexity of its soft decoding algorithm.

The following proposed algorithm replaces the check node equation of the BP algorithm by the simple check node equation of the Luby decoding algorithm. It also exploits the soft values available at the receiver by iteratively updating the value of the LLR available at the variable node $V1$ from Fig. 1 leading to the correct convergence of the algorithm.

Algorithm:

Define: j,n = 1,2,….Degree of the check node C
i,k = 1,2,….Degree of the variable node $V2$

Referring to Fig. 1, LLRs are first received at Variable Node $V1$ as described in (2).

1) The LLR are then mapped where any positive value is replaced by logic '0' and any negative value is replaced by logic '1':

$$c_n = \begin{cases} 0, & if\ sign(LLR) = + \\ 1, & if\ sign(LLR) = - \end{cases} \quad (7)$$

2) At check node C, the check node equation computes the value to be sent to $V2$ by xoring the codeword C and the message received from variable node $V2$:

$$t_{n\to k} = c_n \oplus m_1 \oplus m_2 \oplus m_i \oplus \ldots \quad for\ i \neq k \quad (8)$$

3) At the variable node $V2$ bits are received from the edges of the check node equation,:

$$h_{k\to n} = \begin{cases} 0, & if\ majority\ t_{j\to k} = 0 \\ 1, & if\ majority\ t_{j\to k} = 1 \end{cases} \quad for\ j \neq n \quad (9)$$

4) At check node C, all bits received from variable node $V2$

are xored.

$$U_n = h_{1\to n} \oplus h_{2\to n} \oplus h_{i\to n} \oplus \ldots \ for\ i = k \qquad (10)$$

A weight W is defined where W is set to a value X if the previously calculated U equals '0' or -X if U is equal '1'.

$$W = \begin{cases} X, & if\ U_n = 0 \\ -X, & if\ U_n = 1 \\ 0, & Otherwise \end{cases} \qquad (11)$$

The proper value of the constant X will be obtained from simulation in the section below.

1) The LLR in $V1$ is then updated by the weight W:

$$LLR = LLR + W \qquad (12)$$

2) After the operation is repeated until a maximum number of iteration is reached, the message is then computed at $V2$:

$$m_k = \begin{cases} 1, & if\ majority\ t_{j\to k} = 1 \\ 0, & if\ majority\ t_{j\to k} = 0 \end{cases} \quad for\ j = n \qquad (13)$$

The advantage of the proposed algorithm is as follow:

1) The weight in the above algorithm iteratively corrects the LLR used in the decoding process to get the information symbols.
2) Check node equation has been simplified for adequate hardware implementation.

IV. Simulation Results

In order to evaluate the performance of the system, the encoder-decoder system is implemented in MATLAB. The encoded bits are transmitted using BPSK over an AWGN channel. Simulations were carried out with information bits K=1024. The number of iterations is limited to 50.

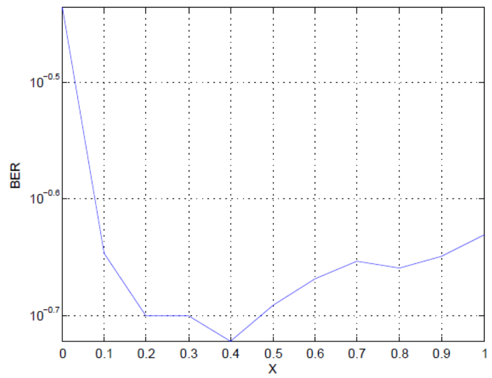


Fig. 3. BER versus X at Eb/No=0 dB.

Fig. 3 investigates the value of the constant X that will lead to the lower bit error rate for the degree distribution in [11]. After sweeping over a value of X ranging from 0 to 1, X=0.4 is an appropriate choice at Eb/No=0dB. The simulation process has been repeated for a range of Eb/No from 0dB to 5dB. X ranging from 0.3 to 0.5 can be accepted as adequate values that yields to the minimum BER.

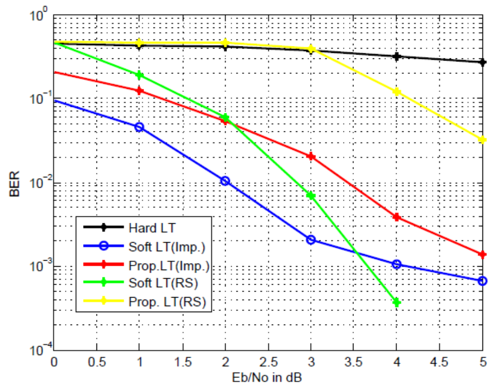


Fig. 4. BER versus Eb/No in dB.

In Fig. 4, the performance for hard decision, soft-decision and the proposed decoder at rate ½ is compared. Soft-based algorithm and the proposed algorithm are compared using two different degree distribution, the Robust Soliton distribution with parameters c=0.3 and δ=0.99 [3] and the improved degree distribution [11] optimized for rate 1.2 shown in (2).

Both decoders, soft decision and the proposed one show better performance using the improved degree distribution [11]. But for higher SNR, the soft decision decoder using Robust Soliton shows very low error floor compared to the improved distribution. The proposed one using Robust Soliton performs poorly.

Using the distribution from [11] the proposed algorithm performs better than hard-decision at all Eb/No due to the use of the soft inputs. It also performs better than the Soft Decision decoder using Robust Soliton at low SNR. Its performance is worse than the soft decision due to the simplified check node equation.

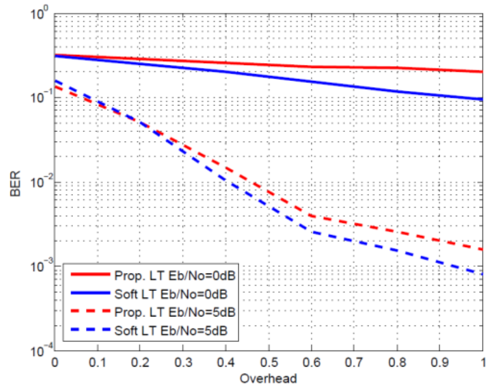


Fig. 5. BER versus overhead.

Finally, Fig. 5 shows the BER versus the overhead at SNR=0dB and SNR=5dB for the soft decision decoder and the proposed algorithm with the overhead ranging from 0 to 1. It is obvious that the performance of both decoders improves with larger overhead. An error floor can be observed as mentioned in [4].

V. Conclusion

This paper presents a new algorithm for LT decoder which merges between soft-decision and hard-decision decoding techniques. A matlab simulation is performed to evaluate the proposed algorithm for two different degree distributions.

The simulation results show that the BER of the proposed algorithm at high SNR is comparable with the soft decision algorithm. At low signal to noise ratio, performance degradation is observed. To overcome this performance degradation, we can consider LDPC concatenation for raptor decoders in future work.

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