The Applications and Simulation of Adaptive Algorithms in MATLAB

T. Sreekanth Rao, B. Prathyusha, P. Nagarjuna Reddy, and K. Raja Rajeswari

Abstract—**Adaptive filters are widely being applied in various fields, including Noise Cancellation, Line Enhancement and System Identification. The most common implementation of Adaptive filters utilizes the time-domain and transform-domain based adaptive filters employing various algorithms to adjust filter coefficients. In this paper, the real time Demonstration of adaptive algorithms has been synthesized within Matlab and a Graphical User Interface (GUI) was developed to the end user with different applications. Also, Least Mean Square, Leaky Least Mean Square, Normalized Least Mean Square, Recursive Least Square, Sign-Error and Sign-Sign adaptive algorithms have been studied and applied. The results have been presented for some cases for specific simulation length and applications.**

Index Terms—**Adaptive noise canceller, adaptive filter, system identification, channel simulator.**

I. Introduction

In recent years, a great progress has been made in the theory, methods and applications of adaptive signal processing. The objective is to introduce the basic concepts underlying the area of adaptive filtering from a philosophical and theoretical point of view. It reviews the concepts of linear adaptive filters and provides an indication of some of the many, choices available by way of adaptation algorithms. The basic application structures are introduced and some of the classical "real-world" applications linked to these. Linear adaptive systems are discussed. Adaptive filters are widely being applied in various fields, including Noise Cancellation, Line Enhancement and System Identification. The most common implementation of Adaptive filters utilizes the time-domain and transform-domain adaptive filters.

Adaptive filters are normally defined for problems such as electrical noise cancelling where the filter output is an estimate of a desired signal. In control applications, however, the adaptive filter works as a controller controlling a dynamic system containing actuators and amplifiers etc. An adaptive filter is a filter that self-adjusts its transfer function according to an optimizing algorithm. Because of the complexity of the optimizing algorithms, most adaptive filters are digital filters that perform digital signal processing and adapt their performance based on the input signal. By way of contrast, a non-adaptive filter has static filter coefficients (which collectively form the transfer function). Adaptive coefficients are required since some parameters of the desired processing operation (for instance, the properties of some noise signal) are not known in advance. In these situations it is common to employ an adaptive filter, which uses feedback to refine the values of the filter coefficients.

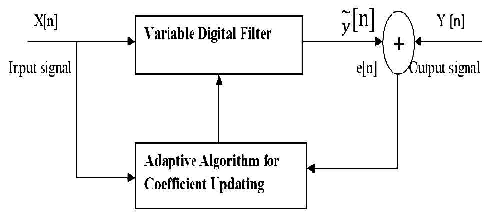

Fig. 1. General form of adaptive filter

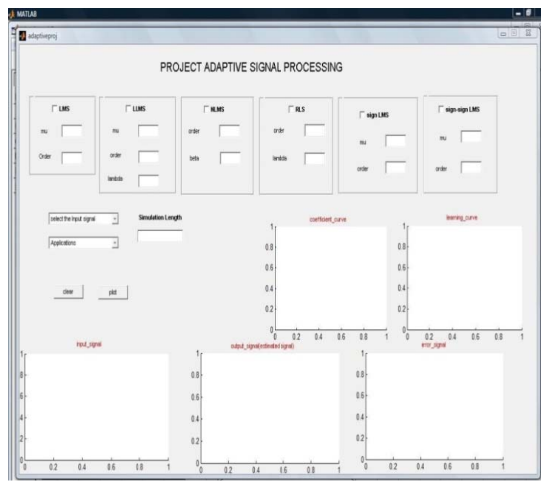

Fig. 2. GUI template: simulator model

The block diagram, shown in the following Fig.1, serves as a foundation for particular adaptive filter realizations, such as Least Mean Squares (LMS) and Recursive Least Squares (RLS), etc. The idea behind the block diagram is that a variable filter extracts an estimate of the desired signal. All the results can be observed using Simulation template called Matlab GUI. A GUI is a graphical display in one or more windows containing controls, called components that enable a user to perform interactive tasks. In this particular template Fig. 2 the user can select the algorithm which he requires, can select the appropriate application and can acquire desired simulation results. The simulator has six adaptive algorithms to select in which each has been distinguished with individual panels. The panel consists of edit boxes in which the user can assign the input values for the variables of that particular

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T. Sreekanth Raot is with the Department of Signal Processing, School of Engineering, Blekinge Institute of Technology, SE- 371 79, Karlskrona, Sweden (e-mail: rao.sreekanth.sp@gmail.com).

B. Prathyusha is with Department of ECE, Malla Reddy College of Engineering and Technology, Hyderabad,Andhra Pradesh,India -500014 (e-mail: prathyusha.signalprocessing@gmail.com).

P. Nagarjuna Reddy is with Department of EEE, Kakatiya Institute of Technology and Science, Warangal 506015, India (e-mail: nagarjunreddyp@gmail.vom).

K Raja Rajeswari is with the Department of ECE, Andhra University College of Engineering, Visakhapatnam, India 530003 (e-mail: krrauv@yahoo.com).

algorithm. There are different types of inputs (here stationary and Non-stationary) and three different types of applications. There are two pop-up menus, from which the user can select the type of input and application. The user can select any application with any desired input signal. There are five axes in which the user can observe five different output wave forms as Input signal, estimated signal, Learning curve, Co-efficient curve, Error signal. The user can also assign the simulation length. One can estimate the signal performance based on these different algorithms with different applications.

II. Algorithms for Adaptive Filters

The algorithm is the procedure used to adjust the adaptive filter coefficients in order to minimize a prescribed criterion. The algorithm is determined by defining the search method (or minimization algorithm), the objective function and the error signal nature. The choice of the algorithm determines several crucial aspects of the overall adaptive process, such as existence of sub-optimal solutions, biased optimal solution, and computational complexity.

A. The Leaky Mean Square (LMS) Algorithm

The LMS is a search algorithm in which a simplification of the gradient vector computation is made possible by appropriately modifying the objective function. The LMS algorithm, as well as others related to it is widely used in various applications of adaptive filtering to its computational simplicity. The convergence characteristics of the LMS algorithm are examined in order to establish a range for the convergence factor that will guarantee stability. The convergence speed of the LMS is shown to be dependent on the Eigen value spread of the input signal correlation matrix. The main features that attracted the use of the LMS algorithm are low computational complexity, proof of convergence in stationary environment, unbiased convergence in the mean to the Wiener solution, and stable behavior when implemented with finite-precision arithmetic. The optimal (Wiener) solution is given by

$$W_o = R^{-1} r_{dx} \quad (1)$$

where $R = E[X(k)X^T(k)]$ & $r_{dx} = E[d(k)\,X(k)]$, assuming that *d(k)* and ***X(k)*** are jointly wide –sense stationary. The update equation for the least mean square (LMS) is given by

$$w(k+1) = w(k) + \mu e(k)X(k) \quad (2)$$

Here e(k)**X(**k) is the estimate of the gradient of $\varepsilon(k)$ and μ is known as the step size and it is computed as[1]

$$0 < \mu < \frac{2}{(p+1)E|e(k)|^2} \quad (3)$$

A simple and effective algorithm that does not require any ensemble averages to be known. For wide-sense stationary processes, the LMS algorithm converges in the mean if the step size is positive and no larger than $2/\lambda_{max}$, and it converges in the mean-square if the step size is positive and no larger than $2/tr\ (R_x\ )$ [1].

B. The Leaky LMS (LLMS) Algorithm

The main difference between LLMS and common LMS algorithm is the introduction of γ into the autocorrelation matrix of the input signal ***X(k)***. One of the main situations in this algorithm is when the autocorrelation matrix of the filter input signal is ill conditioned, which means it has Eigen values close to zero. From the point of formula one can observe the following differences.

The update equation for the LLMS algorithm is given by [1]

$$w(k+1) = (1-\mu)w(k) + e(k)X(k) \quad (4)$$

Here γ is the leakage coefficient which has limits

$$0 < \gamma << 1 \quad (5)$$

Through proper choice of positive leakage parameter ?, excess parameter drift can be avoided. Though LLMS has very low implementation complexity, it applies filter tap leakage indiscriminately thus biasing *W(k)* and increasing mean square error (MSE).The leaky LMS algorithm is useful in overcoming the problems that occur when the autocorrelation matrix of the input process is singular [1].

C. Normalized LMS (NLMS) Algorithm

To increase the speed of the LMS algorithm without using the estimates of the input signal correlation matrix, a variable convergence factor is a natural solution. The normalized LMS algorithm usually converges faster than the LMS algorithm since; it utilizes a variable convergence factor aiming at the minimization of the instantaneous output error. The updating equation of the LMS algorithm can employ a variable convergence factor μ_k in order to improve the convergence rate. In this case the updating formula is expressed as [1]

$$w(k+1) = w(k) + 2\mu_k e(k)X(k) \quad (6)$$

where μ_k must be chosen with the objective of achieving a faster convergence. A possible strategy is to reduce the instantaneous squared error as much as possible. The motivation behind this strategy is that the instantaneous squared error is a good and simple estimate of the mean square error (MSE).

$$\mu_k = \frac{\beta}{X^T(k)X(k)} \quad (7)$$

Using the variable convergence factor, the updating equation for the LMS algorithm is given by [4]

$$w(k+1) = w(k) + \frac{e(k)X(k)}{X^T(k)X(k)} \quad (8)$$

Using a fixed convergence factor μ_n is introduced in the updating formula in order to control the misadjustment, since all the derivations are based on the instantaneous values of the squared errors and not on the MSE. Also a parameter ε should be included in order to avoid large step sizes when $X^T(k)X(k)$ becomes small. The coefficient updating equation is then given by [4]

$$w(k+1) = w(k) + \frac{\beta e(k)X(k)}{\varepsilon + X^T(k)X(k)} \quad (9)$$

The resulting algorithm is called the NLMS algorithm. It simplifies the selection of the step size to ensure that the coefficients converge [1].

D. Recursive Least Square (RLS) Algorithm

The RLS algorithm utilizes least squares approach, it minimizes least square error. Strictly speaking it minimizes error between least squares and mean squares. RLS algorithm minimizes weighted least square error at time 'n' given by the equation [1]

$$\varepsilon(n) = \sum_{i=0}^{n} \lambda^{n-1} |e(i)|^2 \quad (10)$$

Least-squares algorithms aim at the minimization of the sum of the squares of the difference between the desired signal and the model filter output. When new samples of the incoming signals are received at every iteration, the solution for the least squares problem can be computed in recursive form resulting in the RLS algorithms. This algorithm is known to pursue fast convergence even when the Eigen value spread of the input signal correlation matrix is large. This algorithm has excellent performance when working in time varying environments. RLS algorithm is computationally more complex than the LMS adaptive filter, for wide-sense stationary processes [1].

E. Sign Error Algorithm

Sign-error algorithm is one way of the simplifications to reduce the complexity. The Sign-error algorithm is equivalent to the LMS algorithm with a step size that is inversely proportional to the magnitude of the error. In this algorithm, the LMS coefficient update equation is modified by applying the sign operator to the error *e(k)*.Assuming the *X(k)* and *d(k)* are real valued processes , the signal error algorithm is [1]

$$w(k+1) = w(k) + \mu \mathrm{sgn}\{e(k)\}X(k) \quad (11)$$

$$\text{where} \quad \mathrm{sgn}\{e(k)\} = \begin{cases} 1 & ; \; e(k) > 0 \\ 0 & ; \; e(k) = 0 \\ -1 & ; \; e(k) < 0 \end{cases} \quad (12)$$

The simplification in the Sign-error algorithm comes when the step size is chosen to be a power of two, $\mu = 2^{-l}$. In this case, the coefficient update equation may be implemented using *p+1* data shifts instead of *p+1* multiplies. When compared to the LMS algorithm, the Sign-error algorithm uses a noisy estimate of the gradient. The change observed when replacing *e(k)* with the sign of the error is that only the change in magnitude of the correction that is used to update $w(k)$ and does not alter the direction [5]. If μ is constrained to the form $\mu = 2^m$, only shifting and addition operations are required. The update mechanism is degraded, compared to LMS algorithm. The steady state error will increase, the convergence rate decreases.

F. Sign – Sign Algorithm

Sign-Sign algorithm is another way of the simplification to reduce the complexity. This algorithm is equivalent to the LMS algorithm with a step size that is inversely proportional to the magnitude of the error as well as data. In this algorithm, the LMS coefficient update equation is modified by applying the sign operator to both the error *e(k)* and data *X(k)*.Assuming the *X(k)* and *d(k)* are real valued processes , the signal error algorithm is [1]

$$w(k+1) = w(k) + \mu \mathrm{sgn}\{e(k)\}\mathrm{sgn}\{X(k)\} \quad (13)$$

$$\text{Where} \quad \mathrm{sgn}\{e(k)\} = \begin{cases} 1 & ; \; e(k) > 0 \\ 0 & ; \; e(k) = 0 \\ -1 & ; \; e(k) < 0 \end{cases} \quad (14)$$

Unlike any other algorithm, whether or not, sign-sign LMS converges is independent of the magnitude of the step size. The sign-sign algorithm convergence time depend on the noise variance, initial and tolerable magnitudes of the mean weight misalignment, and the Eigen values of the input covariance matrix [5]. This algorithm is faster than sign LMS algorithm.

III. Applications for Adaptive Algorithms

A. Noise Cancellation

One of the most common practical applications of adaptive filters is noise cancellation. The Application of Adaptive Noise Canceller (ANC) is to remove noise from a signal. The concept of ANC, an alternative method of estimating signals corrupted by additive noise or interference [5] as shown in fig. 2, the method uses a primary input d[n] containing the corrupted signal and a reference input x[n] containing noise correlated in some unknown way with the primary noise. The reference input is adaptively filtered and subtracted from the primary input to obtain the signal estimate [6].

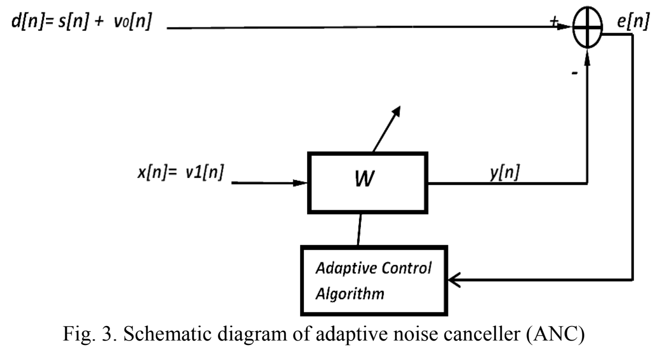

Fig. 3. Schematic diagram of adaptive noise canceller (ANC)

B. Adaptive Line Enhancement (ALE)

The ALE was introduced by Widrow et al for use in adaptive signal processing as a spectral estimation technique. The structure of an ALE or adaptive notch filter is illustrated in fig. 4. After digitalizing, the input signal *x[n]* composed of a harmonic signal plus broadband background noise; goes into two channels. One is treated as the expected signal to an adaptive filter. The other goes through a decorrelation delay Δ and then acts as a reference input signal to the adaptive filter. The adaptive filter will adjust its own weights according to the Adaptive algorithm.

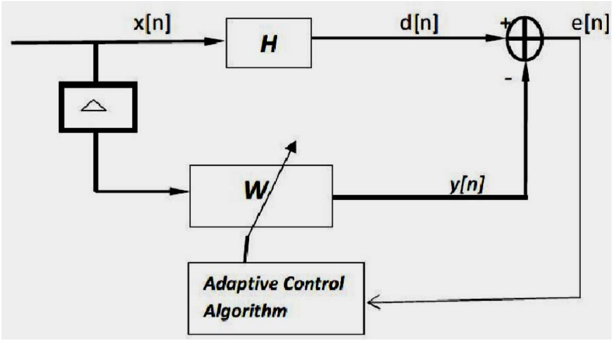

Fig. 4. Schematic diagram of adaptive line enhancer (ALE)

After the optimum processing of the adaptive iteration, the output *y[n]* of the ALE system will suppress the wide band noise, and meanwhile enhance the harmonic signal [2].

C. System Identification

System Identification is experimental approach to the modeling of a process for a channel. In the class of applications dealing with identification, an Adaptive filter is used to provide a linear model that represents the best replacement to unknown channel. The channel and the adaptive filter are driven by same input. The channel output supplies the desired response for the adaptive filter. If the channel is dynamic in nature, the model will be time varying. The schematic block diagram shown in fig. 5, gives the model. The distinguishing characteristic of the system identification application is that the input of the adaptive filter is noise free and the desired response is corrupted by additive noise that is uncorrelated with the input signal. In signal-processing applications, the goal is to obtain a good estimate of the desired response according to the adopted criterion of performance [4].

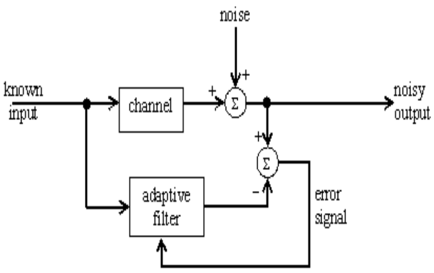

Fig. 5. Schematic diagram of system identification

IV. Simulation Results

The following are the simulation results for the three different applications. The top two axes are

(i) co-efficient curve (ii) learning curve. The remaining three axes are (iii) input signal (iv) estimated/ output signal (v) error signal. In this we have shown different simulation results. Choosing different applications for one particular algorithm, i.e., for LMS Algorithm. Fig. 6 shows the results are obtained for the application Noise Canceller. The variable values are as follows: μ =0.02, order =10, simulation length =1000.

A. LMS Algorithm with Noise Cancellation as Application

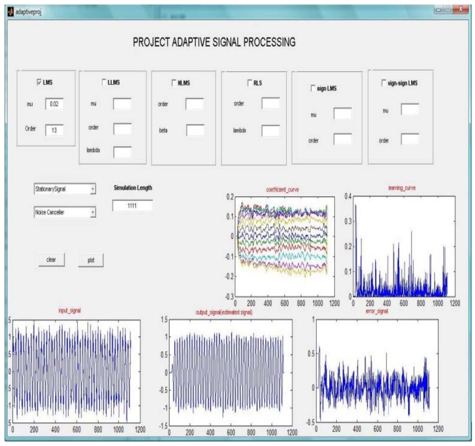

Fig. 6. Output waveforms: LMS algorithm with noise canceller as application

B. LLMS Algorithm with ALE as Application

Fig. 7 shows the results are obtained for the application ALE. The variable values are as follows: μ =0.02, order =10, γ= 1 & simulation length =1000.

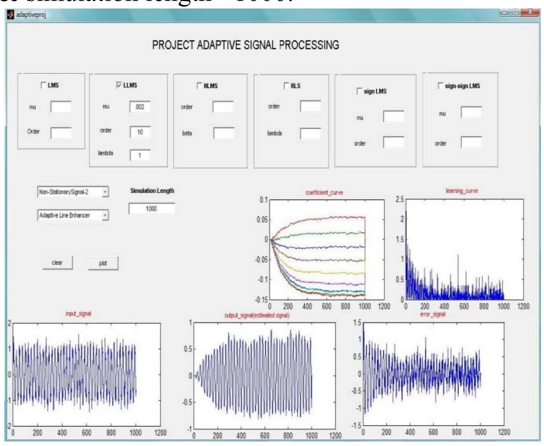

Fig. 7. Output waveforms: LLMS algorithm with ALE as application

C. NLMS Algorithm with System Identification as Application

Fig. 8 shows NLMS is selected to work and β is set to 1 and the order is set to 25. A Stationary signal is selected for System Identification and the output is taken to be the learning curve. The simulation length is chosen as 1000.

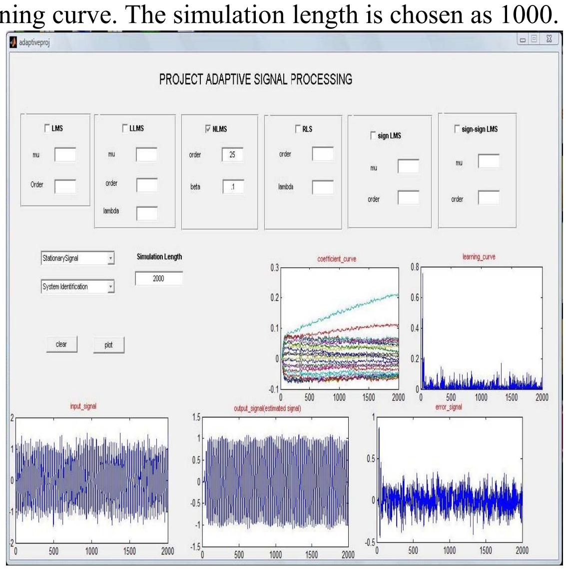

Fig. 8. Output waveforms: NLMS algorithm with system identification as application

D. RLS Algorithm with ANC as Application

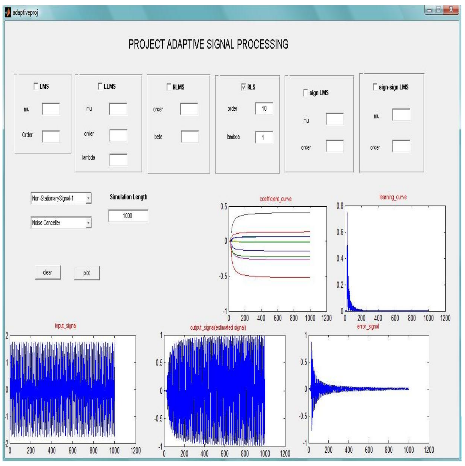

Fig. 9. Output waveforms: RLS algorithm with ANC as application

Fig. 9 shows RLS is selected to work and γ is set to 1 and the order is set to10. A Non-Stationary signal is selected for ANC and the output is taken to be the learning curve. The simulation length is chosen as 1000.

E. Sign Error Algorithm with ALE as Application

Fig.10 shows Sign-Error is selected to work and μ is set to 0.001 and the order is set to 10. A Non-Stationary signal is selected for ALE and the output is taken to be the learning curve. The simulation length is chosen as 1000.

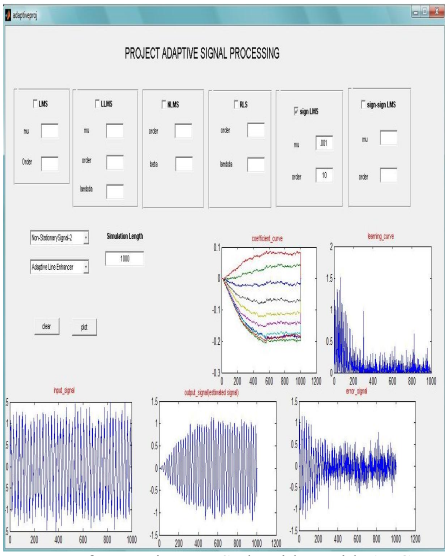

Fig. 7. Output waveforms: sign LMS algorithm with ALC as application

F. Sign – Sign Error Algorithm with System Identification as Application

Fig.11 shows Sign-Sign Error is selected to work and μ is set to 0.001 and the order is set to 10. A Non-Stationary signal is selected for system identification and the output is taken to be the learning curve. The simulation length is chosen as 1000.

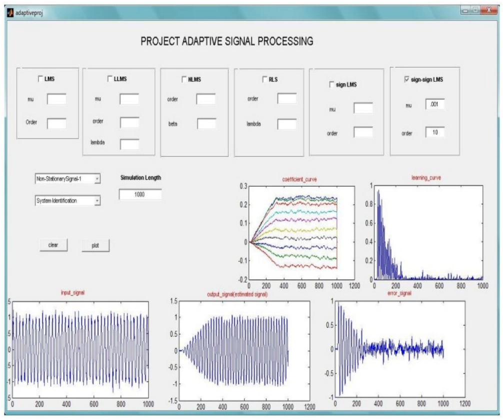

Fig. 8. Output waveforms: sign-sign error algorithm with system identification as application

V. Conclusion

The implementation of various algorithms successfully achieved system identification and noise cancellation. The effect of varying different parameters in the algorithm is also observed.

A performance analysis of adaptive filter algorithms based on adaptive weight update equation is presented in stationary and non-stationary environments. It should be noted that all results presented here for the different algorithms with different applications are demonstrated and the end user can select an appropriate parameter for the desired application.

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Sreekanth Rao Tangellapally born in Warangal, India in 1987. He received the Bachelor degree in Electronics and Instrumentation Engineering from Kakatiya University, Warangal, in 2008. The M.S. in Electrical Engineering, with emphasis on Signal Processing at Blekinge Institute of Technology, Karlskrona, Sweden (at present) and M.Tech. Degree from Andhra University, Visakhapatnam, India, in 2011. He has received academic awards and publications in reputable IEEE Conferences and Journals. He is author of the textbook Fingerprint Recognition System published by *VDM Verlag Dr. Müller e.K.* His areas of interests are Image Processing and Bio-medical Signal Processing.

B Prathyusha born in Warangal, India in 1987. She received the Bachelor degree in Electronics and Communication Engineering from Jawaharlal Technological University (JNTU) in 2009. She is currently undergoing a Master's program in Systems & Signal Processing at Malla Reddy College of Engineering & Technology, Hyderabad. She has received academic awards and publications in reputable International Conferences and Journals. Her areas of interests are Speech Signal Processing and Bio-medical Signal Processing.

Nagarjuna Reddy completed his Bachelor degree in Electrical & Electronics Engineering from University college of Engineering, JNTU, Hyderabad and Master's Program in High Voltage Engineering from University college of Engineering, JNTU, Kakinada He is currently pursuing his research in JNTU, Hyderabad. His areas of interests are Gas insulated Substations, MATLAB applications in Electrical and Electronics.

Prof.K. Raja Rajeswari obtained her B.E., M.E. and Ph.D. degrees from Andhra University, Visakhapatnam, India in 1976, 1978 and 1992 respectively. Presently she is professor in the Department of Electronics and Communication Engineering, Andhra University. She is Dean for Quality Assurance, Andhra University College of Engineering. She has published over 100 papers in various National, International Journals and conferences. She is author of the textbook Signals and Systems published by PHI. She is co-author of the textbook Electronics Devices and Circuits published by Pearson Education. Her research interests include Radar and Sonar Signal Processing, Wireless Communication Technologies. She has guided twelve Ph.D.s and presently she is guiding fifteen students for Doctoral degree. She is immediate past chairperson of IETE, Visakhapatnam Centre. Present she is Governing Council Member of IETE, New Delhi. She is recipient of prestigious IETE Prof SVC Aiya Memorial National Award for the year 2009, Best Researcher Award by Andhra University for the year 2004 and Dr. Sarvepalli Radhakrishnan Best Academician Award of the year 2009 by Andhra University. She is expert member for various national level academic and research committees and reviewer for various national/international journals.