Signal Processing and Time-frequency Analysis Application in Portfolio

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***Abstract*—Studies show that most actively managed mutual funds struggle to beat the market, driving an increase in the popularity of index investing. Index investing instruments, including index funds and ETFs, aim to track market performance. This study pursues both tracking error minimization and excess return maximization, two conflicting objectives, to construct a portfolio. The fuzzy model that track indexes is developed and discussed.**

***Index Terms*—portfolio, model, index.**

I. Introduction

Investors purchase mutual funds because they believe that diversification brings lower risks and professional management brings higher returns than individual stocks (Gruber [1996]). The professional managers of actively managed mutual funds try to outperform the market by superior stock-picking techniques. But the truth is ironic. Regardless of investment styles (growth vs. value, small-cap vs. large-cap), most actively managed mutual funds cannot beat the market, and those top-rated mutual funds seldom beat the market annually (Bogle [1998]). As Ellis mentioned, "investment managers are not beating the market: The market is beating them." (Ellis [1975]) . Even higher fees don't create higher returns for investors (Gil-Bazo, J. and P. Ruiz-VerdÚ [2009]). More and more investors are turning to index investing.

Another alternative of investing in S&P 500 is to purchase an Exchange-Traded Fund (ETF). Exchange-Traded Fund is a security that tracks the index like an index mutual fund but can be traded just like a stock. The world's first Exchange-Traded Fund, SPDR S&P 500 ETF (AMEX:SPY), is launched in 1993 and managed by State Street Global Advisors. It also tracks one of the most popular indexes in the world, the S&P 500 Index. By the end of 2009, there were 1,800 ETFs trading worldwide, with assets over $1 trillion.

II. Literature review

Gilli and kellezi [2002] proposed a threshold accepting algorithm for index tracking. Jansen and van Dijk [2002] considered the minimization problem of tracking error when the number of stocks is limited in the portfolio. They first decided the set of stocks, and then they optimize the weights of stocks by a standard quadratic programming method.

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Derigs and Nickel [2003] proposed a simulated annealing based metaheuristic. They presented a case study about an investment trust tracking the German DAX30 index. Okay and Akman [2003] applied constraint aggregation and proposed a mixed-integer nonlinear programming problem. They used data set from Hang Seng Indexes. Focardi and Fabozzi [2004] proposed that using clustering as a method for constructing index tracking portfolios. The Euclidean distances between stock price series are used for hierarchical clustering.

Goal programming (G.P.) was originally developed by Charnes and Cooper (1957). GP is a mathematical programming technique capable of handling multiple and conflicting objectives and widely applied in situations where conflicting objectives usually result in infeasible solutions (Jones & Tamiz, 2010). Wu et al. (2007) used goal programming for the bi-objective index tracking model to maximize excess returns and minimize tracking error. Goal programming basically involves establishing goals and then minimizing the deviation between these goals and practical results. A deviational variable d is generally used in a goal programming model. d+ represents the deviation above a goal and d- represents that below a goal.

The real world is characterized by imprecise boundaries. Taking temperature as an example, if a temperature above 30 °is considered "hot", then surely a temperature of 29.9 °is not considered "cold". Rather a temperature of 29.9 °should be recognized as approximating "hot". This vagueness is easily expressed linguistically. However, how best to quantify vagueness remains unclear. Zadeh (1965) proposed fuzzy set theory to resolve problems that are imprecise rather than random. A fuzzy set is a set of elements with corresponding membership grades. Specifically, a fuzzy set is defined by a membership function which assigns each element a membership grade. The membership grade ranges between zero and one, with the highest grade being one, and the lowest being zero. This investigation used some denotations from (Zadeh, 1965) to obtain a more detailed explanation. Let X denote a set of elements, and $x \in X$. A fuzzy set $\tilde{A}$ in X is defined using a membership function $\mu_{\tilde{A}}(x)$. Meanwhile, $\mu_{\tilde{A}}(x)$ assigns each x a membership grade between zero and one. The nearer $\mu_{\tilde{A}}(x)$ is to one, the higher the membership grade of x in $\tilde{A}$.

Bellman and Zadeh (1970) discussed decision-making in a fuzzy environment. In a fuzzy environment, an imprecise goal is described as a fuzzy goal. For explanation, this investigation used an example given by Bellman and Zadeh

(1970). In the example, the fuzzy goal G is expressed as "x should substantially exceed 10", and can be represented using a membership function, as follows:

$$\mu_G(x) = \begin{cases} 0 & ,x < 10 \\ \left(1 + (x-10)^{-2}\right)^{-1} & ,x \geq 10 \end{cases} \qquad (1)$$

III. Fuzzy Model and Performance Measurement

This study used the membership function to describe decision-maker preferences. In this study, tracking error minimization and excess return maximization are both objectives of the index tracking problem for decision-makers. Rather than linear membership function, this study used a nonlinear exponential membership function able to flexibly express decision-maker preferences. Nonlinear curves provide potential benefits in relation to various perceptions of real-world decision-makers. This study used notations from Kwang-Jae & Dennis (1998) to explain the form of nonlinear membership function used

$$\mu_g(x) = \frac{e^d - e^{d\left(\frac{x-x_u}{x_l-x_u}\right)}}{e^d - 1}, d \neq 0 \qquad (2)$$

$\mu_g(x)$ is an concave nonlinear exponential decreasing membership function for the goal g on x. e is commonly defined as the base of the natural logarithm, and d is a constant which does not equal zero. x lies between x_l and x_u. Moreover, x_l is the lower bound of decision-maker satisfaction. When x equals x_l, $\mu_g(x)$ achieves its minimum value of zero. x_u is the upper bound of decision-maker satisfaction. When x equals x_u, $\mu_g(x)$ achieves its maximum value of one. Kwang-Jae and Dennis used the function in (18) for dual response surface optimization (Kwang-Jae & Dennis, 1998), and moreover this function has been proven able to express human preferences reasonably and flexibly. ((Kirkwood & Sarin, 1980),(Moskowitz, 1993)).

Tracking error is commonly used to measure the risk of an index tracking portfolio. The tracking error measures the difference between the return of an index tracking portfolio and that of its benchmark index (Meade & Salkin, 1989). One of the objectives of this study is to minimize the tracking error of the index tracking portfolio. The formula used to calculate tracking error takes the form of the standard deviation, as follows:

$$Tracking\ Error = \sqrt{\frac{\sum_{t=1}^{n}\left(r_t^p - r_t^b\right)}{n}} \qquad (3)$$

Yang et al. (1991) formulated their fuzzy goal programming model using min-operator. Let G_i represent the i-th fuzzy goal and μ_i denote the satisfaction level of the i-th goal. Yang et al. then aggregated the object function using the min-operator, as shown in the following equation:

$$\begin{aligned} &Maximize && \lambda \\ &subject\ to && \lambda \leq \mu_i, i = 1,2,\ldots,n \\ &&& \mu_j, j \neq i, \\ &&& i, j \in \{1,\cdots,n\} \end{aligned} \qquad (4)$$

IV. Conclusion

First, this research is to propose a new approach to the academic area of enhanced index tracking based on goal programming and fuzzy theory application. Enhanced index tracking is considered as a bi-objective problem. Goal programming is suitable for handling the conflicting objectives, and fuzzy theory is suitable for describe the imprecise feature of the financial environment.

Second, this research is to provide an alternative method for constructing an enhanced index portfolio in Taiwan fund industry. The imprecise decision can be estimated by fuzzy theory reasonably, and the trade-off for risk and return managed by the manager is described by goal programming suitably.

Last, it is attractive to examine the model in the time period including the global financial recession. Index investing concept is accepted increasingly these years, and enhanced index investing comes with the tide of fashion. It deserves examined that whether an enhanced index product is still a robust investment choice in the long term or not.

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