Subspace Based Face Recognition on Hypercomplex Field

Chengzhang Wang and Xiaoming Bai

Abstract—**Color information is very important cues for recognition task. In this paper, a novel color face recognition approach based on subspace is proposed. The approach employs hypercomplex to encode color information of different channels simultaneously. Hypercomplex matrix decomposition is then used to construct feature extraction approach. To improve learning efficiency of the algorithm, 3D active deformable model is exploited to generate virtual color face images. Experimental results on CMU PIE database verify the effectiveness of the proposed approach.**

Index Terms—**Hypercomplex, color face, face recognition**

I. INTRODUCTION

Face recognition is one of the most challenging topics in both pattern recognition and computer vision areas. As two-dimension grey scale image can be easily represented by a single vector or a matrix, grey scale information of face images is utilized to accomplish recognition task [1]. However, color information of face image provides more cues for recognition than grey scale ones [2]. Recently, more and more researchers exploit color information of face images to improve the performance of recognition algorithms [3, 4].

In some research works, color face images is first converted into monochromatic grey scale ones, and then facial features are extracted based on these grey scale images [5, 6]. But the conversion loses a lot of color information of face images which is unfavorable to recognition task. In some research works, three color channels of face image are treated respectively, each color channel is like a monochromatic one. Final result may be achieved by fusing results of the three channels [7]. Whereas, information of different color channels are utilized separately for recognition would destroy the structural integrality of the color information and the correlation among them. In other research works, matrix representation model and block diagonal matrix representation model are proposed to denote color face image [8, 9]. These algorithms encode information of different channels simultaneously which to some extent preserves the integrality and the correlation of color information, and improves color face recognition accuracy. In essence, these methods just encode three color channel information into one subject function simultaneously. And they hardly denote color information of each pixel at one element.

In order to denote color image effectively, hypercomplex is introduced into color image representation [10]. As one hypercomplex number is composed of one real part and three imaginary parts, it can represent four components in one number. Based on the hypercomplex model, a lot of color image processing algorithms employ hypercomplex, such as Fourier transforms [11], edge detection [12] and color filters [13].

II. HYPERCOMPLEX MATRIX DECOMPOSITION

A Hypercomlex q is defined as [14]: $q = a + b\mathrm{i} + c\mathrm{j} + d\mathrm{k}$. Where a, b, c, d are real numbers. $\mathrm{i}, \mathrm{j}, \mathrm{k}$ are imaginary units, and they satisfy: $\mathrm{i}^2 = \mathrm{j}^2 = \mathrm{k}^2 = -1$, $\mathrm{ij} = -\mathrm{ji} = \mathrm{k}$, $\mathrm{jk} = -\mathrm{kj} = \mathrm{i}$, $\mathrm{ki} = -\mathrm{ik} = \mathrm{j}$. a is called real part of hypercomlex q and $b\mathrm{i} + c\mathrm{j} + d\mathrm{k}$ is called imaginary part of hypercomlex q. Hypercomlex q can be rewrite according to imaginary multiplication rule as: $q = a + b\mathrm{i} + (c + d\mathrm{i}) \cdot \mathrm{j}$.

Suppose two hypercomplex $q_1 = a_1 + b_1\mathrm{i} + c_1\mathrm{j} + d_1\mathrm{k}$ and $q_2 = a_2 + b_2\mathrm{i} + c_2\mathrm{j} + d_2\mathrm{k}$, then:

$$
\begin{aligned}
q_1 \cdot q_2 \Leftrightarrow & (a_1a_2 - b_1b_2 - c_1c_2 - d_1d_2) + (a_1b_2 + b_1a_2 + c_1d_2 - d_1c_2)\,\mathrm{i} + \\
& (a_1c_2 + c_1a_2 + d_1b_2 - b_1d_2)\,\mathrm{j} + (a_1d_2 + d_1a_2 + b_1c_2 - c_1b_2)\,\mathrm{k}.
\end{aligned}
$$

The conjugate $\bar{q}$ of hypercomplex q is defined as: $\bar{q} = a - b\mathrm{i} - c\mathrm{j} - d\mathrm{k}$. It is deserved noting that hypercomplex multiplication rule does not satisfy commutativity interchangeability, that is $q_1 \cdot q_2 \neq q_2 \cdot q_1$.

For a hypercomplex matrix, if $\overline{A^{\mathrm{T}}} = \bar{A}^{\mathrm{T}} = A$, the matrix A is defined to be a self-conjugate matrix. Any hypercomlex matrix A can be uniquely decomposed as: $A = A_1 + A_2\,\mathrm{j}$. Where A_1, A_2 are complex matrices. For any hypercomlex matrix A, its deducing matrix on complex field is defined as:

$$
A^{\sigma} = \begin{pmatrix} A_1 & -A_2 \\ \overline{A_2} & \overline{A_1} \end{pmatrix}.
$$

If there are a hypercomlex number λ and a hypercomlex vector ξ which satisfy $A \cdot \xi = \xi \cdot \lambda$, λ is said to be right the eigenvalue of A, and vector ξ is said to be the eigenvector according to λ.

If matrix A is a $n \times n$ self-conjugate matrix, its deducing matrix on complex field is a $2n \times 2n$ complex matrix. According to the matrix theory on complex field, there is

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eigenvalue λ of matrix A^{σ}. That is there is a $2n$ -dimension non-zero column vector $\begin{pmatrix}\xi_1\\ \xi_2\end{pmatrix}$ which satisfies:

$$\begin{pmatrix} A_1 & -A_2 \\ \overline{A_2} & \overline{A_1} \end{pmatrix} \cdot \begin{pmatrix}\xi_1\\ \xi_2\end{pmatrix} = \begin{pmatrix}\xi_1\\ \xi_2\end{pmatrix} \cdot \lambda$$

Construct n -dimension column vector on hypercomplex field: $\eta_1 = \xi_1 + \overline{\xi_2} \cdot \mathrm{j}$. As $\begin{pmatrix}\xi_1\\ \xi_2\end{pmatrix}$ is an eigenvector, there is at least one non-zero vector among ξ_1, ξ_2. So η_1 are non-zero column vectors on hypercomplex field. From above results, one can get:

$$A \cdot \eta_1 = (A_1 + A_2 \cdot \mathrm{j})(\xi_1 + \overline{\xi_2} \cdot \mathrm{j}) = (\xi_1 + \overline{\xi_2} \cdot \mathrm{j}) \cdot \lambda = \eta_1 \cdot \lambda$$

Therefore λ is definitely a right eigenvalue of matrix A and η_1 is the corresponding eigenvector.

As $A \cdot \eta_1 = \eta_1 \cdot \lambda$, then $\eta_1^{\mathrm{H}} \cdot A \cdot \eta_1 = \eta_1^{\mathrm{H}} \cdot \eta_1 \cdot \lambda = |\eta_1|^2 \cdot \lambda = \lambda \cdot |\eta_1|^2$. Therefore one can get:

$$\lambda \cdot |\eta_1|^2 = \overline{\lambda} \cdot |\eta_1|^2 .$$

So right eigenvalue λ of self-conjugate matrix A is definitely a real number.

According to Schur triangular decomposition, there is a $n \times n$ unitary matrix W which satisfies:

$$W^{\mathrm{H}} \cdot A \cdot W = \begin{pmatrix} \lambda_1 & q_{12} & \cdots & q_{1n} \\ & \lambda_2 & \cdots & q_{2n} \\ & & \ddots & \vdots \\ & & & \lambda_n \end{pmatrix}.$$

As matrix A is a $n \times n$ self-conjugate matrix, one can get $A = A^{\mathrm{H}}$. Therefore one can get:

$$A = W \cdot \begin{pmatrix} \lambda_1 & & & \\ & \lambda_2 & & \\ & & \ddots & \\ & & & \lambda_n \end{pmatrix} \cdot W^{\mathrm{H}} = W \cdot diag(\lambda_1, \lambda_2, \cdots, \lambda_n) \cdot W^{\mathrm{H}}.$$

It is deserved noting that unitary matrix W contains the eigenvectors of A.

III. Face Recogniton Based on Subspace Approach

A. Color Face Representation

Each pixel of color face image is composed of three color components: R, G and B. To preserve the integrality of color information of different channels, hypercomplex is employed in this paper to encode trichromatic information of each pixel simultaneously. For each $m \times n$ color face image, color information of each pixel (x, y) is encoded as:

$$q_{xy} = 0 + R_{xy}\mathrm{i} + G_{xy}\mathrm{j} + B_{xy}\mathrm{k}.$$

Then a color face image is represented on hypercomplex field as a column vector x:

$$x = (q_{11}, q_{12}, \cdots, q_{mn})^{\mathrm{T}}.$$

B. Facail Feature Extraction

Having construct hypercomplex representation of color face image, subspace theory is explored in this paper for the purpose of feature extraction. According to the subspace theory, constructing subspace of color face space in this paper is to build a color feature subspace in which the variance of the projected data is maximized. Therefore the optimal projection directions can consist into the projection matrix for feature extraction. Constructing subspace is transformed into the following optimal problem: $\max J(e) = tr(M_e)$. Where $tr(M_e)$ denotes the trace of M_e and M_e is the covariance matrix of the projected data of color face images. Let x_k ($1 \le k \le N$) denote the $k-th$ color face in the training set where N is the number of color face samples the training set. According to the theory of statistics, covariance matrix S_t can be computed as:

$$S_t = \frac{1}{N}\sum_{k=1}^{N}(x_k - \overline{x}) \cdot (x_k - \overline{x})^{\mathrm{H}}$$

where $\overline{x} = \frac{1}{N}\sum_{k=1}^{N} x_k$ denotes the average color face. The optimal projection direction is the eigenvector of the covariance matrix. And in addition, the covariance matrix constructed like the above is a self-conjugate hypercomplex matrix. Therefore we can utilize the above hypercomplex matrix decomposition algorithm presented in section II to compute the projection matrix.

IV. Generating of Virtual Color Face Images

As dimensionality of face image is generally high, large amount of images are needed to improve the learning quality and overcome the so-called "SSS" problem. In this paper, 3D active deformable model [15] is employed to generate virtual color face images. 3D active deformable model is built according to the idea of linear object class. Having established full correspondence between scanned faces, 3D active deformable model is constructed using PCA technique. Given an input 2D face image, the 3D face of the specific person is reconstructed through the model matching process. Reconstructed 3D face can be rotated along each direction and projected into two dimensional image plane to generate realistic virtual color face images. Fig. 1 shows some virtual color face image generated by 3D active deformable model on CMU PIE database. Where, left image is the original input one. Right images are some virtual color face images generated by 3D deformable model according to the input one.

Fig. 1. Parts of virtual color face images

V. Experimental Results

In order to verify effectiveness of the hypercomplex matrix decomposition algorithm for color face recognition, experiments are carried out on CMU PIE [16] databases. At the same time, color face recognition method in [8] is evaluated under the same conditions and experimental results are analyzed in this section.

CMU PIE face database contains color face images of 68 subjects under different poses, illuminations and expressions. In our work, we select color faces of each person in the directory "illum" with identification by "*_07.ppm" for our experiments. An approximate frontal color face image is selected as the input one for 3D active deformable model to generate realistic virtual color face images. Reconstructed 3D face is rotated along X direction from left 45^0 to right 45^0, and three angles are randomly selected for projecting to generate virtual color face images. Therefore there are sixteen color face images of each person totally. In our experiment twelve images of each person are randomly selected to constitute the training set. The rest color faces constitute the testing set. All the color face images are normalized to the same size in our work for experiment. Experiments are conducted on each database for six times, the resultant recognition rate is the average of these six results.

Top match score [17] of the two color face recognition approaches are calculated under different ratio of contribution. Fig. 2 illustrates comparison of face recognition of two approaches on CMU PIE database. Where the horizontal axis is the ratio of contribution and the vertical axis is the top match score. In the figure, "New" stands for the color face recognition approach proposed in this paper and "Old" stands for the recognition approach in [8]. From the results, it can be seen that recognition performance of two approaches also improves as the ratio of contribution increases. Whereas, the top match score of color face recognition approach proposed in this paper is obviously higher than that of the recognition method in [8] under the same conditions.

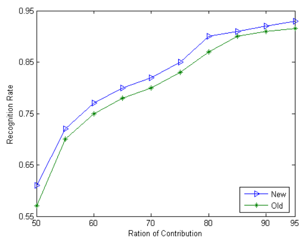

Fig. 2. Top match score on CMU PIE

VI. Conclusions

A new color face recognition approach coined subspace based algorithm using hypercomplex matrix decomposition is proposed in this paper. In order to encode both the color information of different channels, hypercomplex is explored to describe the color face sample. Based on the representation model, hypercomplex matrix decomposition algorithm is developed to compute the feature subspace for feature extraction and the nearest neighborhood classification approach is adopted to identify the color face samples. Experimental results on CMU PIE database show the good performance of the proposed approach.

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